Two-nucleon cluster spectroscopic factors

de Meijer, RJ; Put, LW; Vermeulen, JC

Published in:
Physics Letters B

DOI:
10.1016/0370-2693(81)91136-9

Publication date:
1981

TWO-NUCLEON CLUSTER SPECTROSCOPIC FACTORS: A SEARCH FOR THE BOUND-STATE PARAMETERS

R.J. de MEIJER, L.W. PUT and J.C. VERMEULEN[1]
Kernfysisch Versneller Instituut, Rijksuniversiteit Groningen, 9747 AA Groningen, The Netherlands

Received 23 August 1981

A method is presented to extract relative two-nucleon spectroscopic factors from experimental data with DWBA calculations using a macroscopic form factor. The radius and diffuseness of the macroscopic bound-state well are adjusted such that macroscopic and microscopic form factors are almost identical for all L-values with $L \leqslant 6$. A common area exists in the (r_0, a) plane where this condition is satisfied.

The main difference in the expressions for the cross section of single-nucleon transfer reactions (SNT) and two- (and in general multi-) nucleon transfer reactions (TNT) is the fact that for SNT the dynamical and structure parts factorize, whereas this is not the case for TNT. For SNT the ratio between the experimental and calculated (e.g. in DWBA) cross sections yields, apart from a normalization constant and statistical factors, the spectroscopic factor which can directly be compared with the results of model calculations. To calculate the dynamical part one assumes the transferred particle to be bound to the nucleus represented by a potential well with size parameters which are believed to be well established and a depth adjusted to reproduce the experimental separation energy.

For TNT the shape and magnitude of the angular distribution depend on the coherence properties of the overlap between the initial and final state. A set of model wavefunctions and in particular their coherence properties may be tested by employing the resulting transition amplitudes in a microscopic DWBA calculation and comparing the angular distribution obtained with experimental data. Due to the coherence effects, in general, no unique information on the individual amplitudes of the various configurations can be extracted from this comparison. In multi-nucleon transfer reactions, as α-transfer reactions (see e.g. ref. [1]) it is frequently assumed that the factorization of the cross section may also be carried out and the form factor is then calculated in a macroscopic approach, namely under the assumption that the transferred cluster is structureless.

In this letter the macroscopic approach is applied to (α, d) reactions on ^{28}Si and ^{32}S and to the ^{32}S(d, α)^{30}P reaction. Here the relative intensities of the DWBA cross sections for different L-values appear to be a function of the size parameters of the bound-state well. For all L-values with $L \leqslant 6$ a common area was found in the (r_0, a) plane where the radial dependence of the macroscopic and microscopic form factors is almost identical. The details of the analyses will be presented in a forthcoming publication [2].

In DWBA calculations (e.g. with DWUCK IV [3]) the cross section for the (α, d) or (d, α) reaction is given as

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{J}_{\mathrm{DWBA}} \propto \left| \sum_{LSM} \frac{1}{2L+1} \times \int \mathrm{d}^3 R\, \chi^{-*}(\boldsymbol{k}_\mathrm{f}) F_{JLST}(R)\, Y^{M}_{L}(\hat{R}) \chi^{+}(\boldsymbol{k}_\mathrm{i}) \right|^2 . \quad (1)$$

Here F is the two-nucleon form factor, and $\chi(\boldsymbol{k}_\mathrm{i})$ and $\chi(\boldsymbol{k}_\mathrm{f})$ are the distorted ingoing and outgoing waves.

[1] Present address: CERN, CH-1211 Geneva 23, Switzerland.

0 031-9163/81/0000–0000/$ 02.75

In the microscopic approach two-nucleon transfer amplitudes β_γ, $\gamma = (n_1 l_1 j_1; n_2 l_2 j_2)$, are deduced from the shell-model wave functions. In order to construct the form factor F a transformation is made from individual to centre-of-mass (c.m.) amplitudes $\beta_{\gamma JLST}$ following the Bayman–Kallio method [4]. This method assumes a relative s-motion between the transferred nucleons.

For harmonic-oscillator radial wavefunctions the relation between the quantum numbers for the relative (ν, λ) and c.m. motion (N, Λ) of the cluster and those for the individual nucleons (n_i, l_i) is given by

$$2(\nu + N) + \lambda + \Lambda = 2(n_1 + n_2) + l_1 + l_2. \quad (2)$$

It is commonly assumed that eq. (2) is also valid for Woods–Saxon wavefunctions. Assuming that only two nucleons in a 0s state contribute to the cross section implies that for each L-value ($L = \Lambda$) only one N-value remains. Further assuming no L–S coupling, eq. (1) may be written [2] as an incoherent sum of contributions with different L value

$$\left(\frac{d\sigma}{d\Omega}\right)^J \propto \sum_L |G_L|^2 \sum_M |B_L^M|^2. \quad (3)$$

All structure information represented in c.m. coordinates is contained in G and hence for the calculation of $\Sigma_M |B_L^M|^2$ no further structure information is needed [5]. For a set of bound-state parameters r_0 and a this factor may be calculated as $(d\sigma/d\Omega)^L_{\text{macro}}$ with a DWBA code like DWUCK IV. In the comparison with experimental data one determines, apart from a normalization constant αC, the experimental intensities A_L^2 for each L-value. The experimental cross section for reactions with $\Delta S = 1$ can thus be written as

$$(d\sigma/d\Omega)^J_{\text{exp}} = \alpha C (2J+1)^{-1} \times \{A_L^2 (d\sigma/d\Omega)^L_{\text{macro}} + A_{L+2}^2 (d\sigma/d\Omega)^{L+2}_{\text{macro}}\}, \quad (4)$$

where α is the product of statistical factors.

In the microscopic case the cross section is also a sum of contributions with different L-value. It should be noted here that the Bayman–Kallio method includes relative s-motions other than zero allowing several N-values for a particular L-value. The form factor may be represented as $G_L\, U_L(R)$ where U_L is the normalized form factor used in the calculation of B_L^M and the cross section can be written [2] as

$$(d\sigma/d\Omega)^J_{\text{exp}} = \alpha C (2J+1)^{-1} \times \{G_L^2 (d\sigma/d\Omega)^L_{\text{micro}} + G_{L+2}^2 (d\sigma/d\Omega)^{L+2}_{\text{micro}}\}. \quad (5)$$

The square brackets indicate the cross section calculated with a normalized form factor the shape of which is determined by the transformation from individual to c.m. and relative cluster coordinates.

Any macroscopic form factor which satisfies $U_L(R)^{\text{macro}} = U_L(R)^{\text{micro}}$, yields identical cross sections $(d\sigma/d\Omega)^L$ in eqs. (4) and (5). Then the A_L^2, obtained, e.g. by a fitting procedure to the data, might be interpreted as experimental two-nucleon spectroscpic factors and are related to G_L^2 by $\Omega_{00}^2 A_L^2 = G_L^2$. The factor Ω_{00}^2 [2,5] contains all structure dependence on the light-particle combination and is equivalent to D_0^2 in the usual zero-range approximation for SNT reactions. Such factors are usually absorbed in the normalization constant.

In the following we present a procedure to obtain the r_0 and a values that yield approximately identical microscopic and macroscopic form factors. Macroscopic form factors were generated by taking a deuteron with quantum numbers N, L and J in a Woods–Saxon well with a depth yielding the appropriate binding energy. Those values of r_0 and a were accepted for which the maxima, the minima, and the zero-crossing of the macroscopic form factors

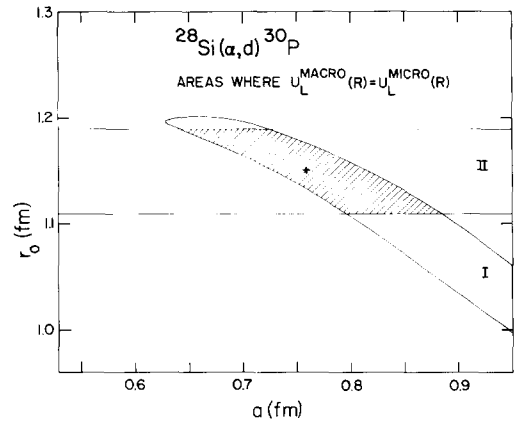

Fig. 1. Areas in the plane formed by the geometrical parameters of the cluster Woods–Saxon well. Region I results from the restriction $U_L^{\text{macro}}(R) = U_L^{\text{micro}}(R)$ for all L-values from $L = 0$ to $L = 6$. Region II results from imposing the well-matching prescription. For details see text. The cross indicates the actually chosen values.

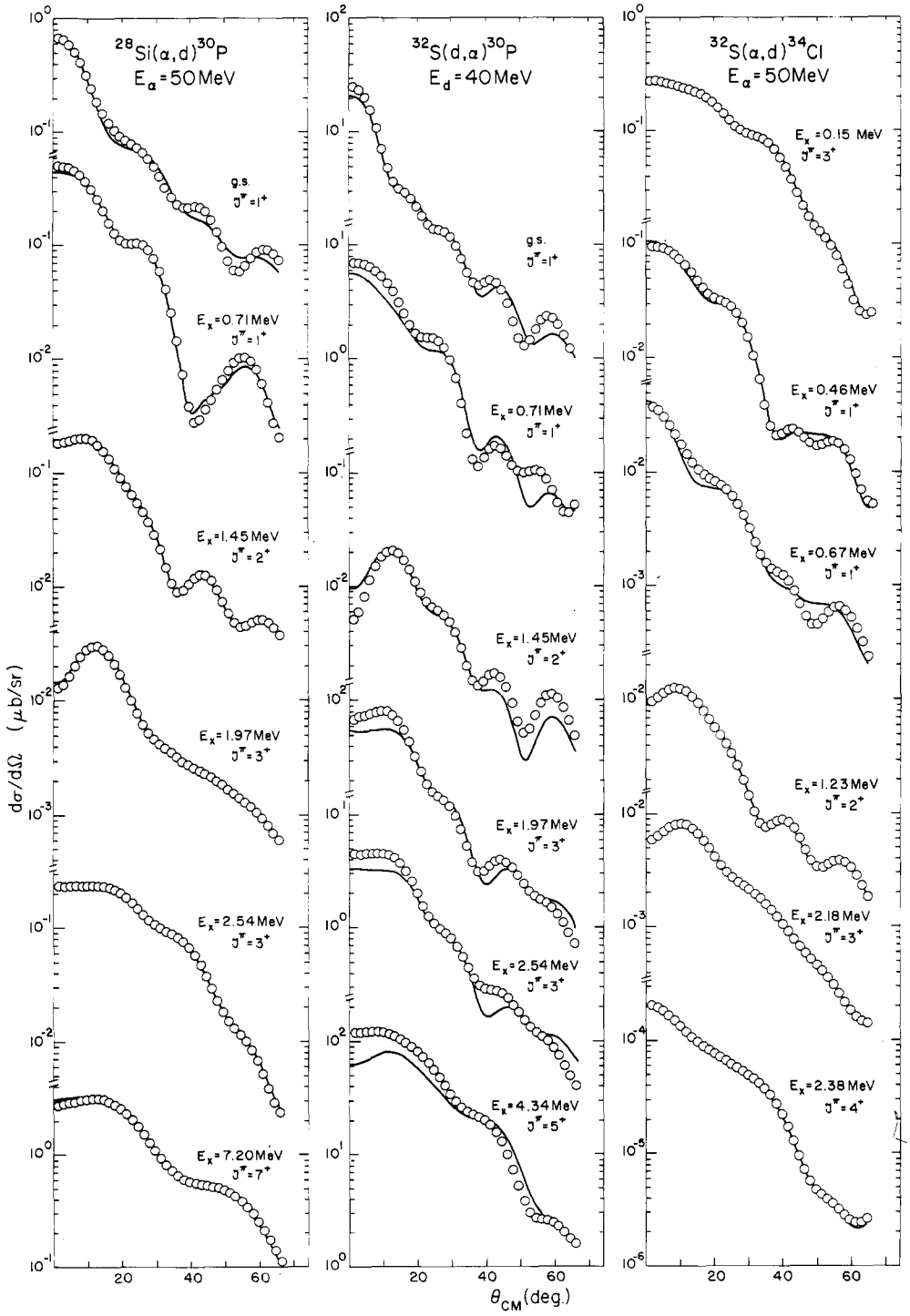

Fig. 2. A comparison of microscopic angular distributions and the corresponding macroscopic ones for a number of states in the (α, d) and (d, α) reactions. The data points represent the microscopic values, the solid line represents the best χ^2-fit to them.

are within 0.1 fm of their microscopic counterparts. For form factors without nodes the same criteria were used for the values at half maximum. With these requirements the shapes of the form factors turned out to be almost identical. It also turned out that the shape of $U_L(R)$ is almost independent of the configuration γ. This not only is a support for the restriction to zero relative s-motion in the macroscopic calculations but also indicates that the results of this search are practically independent of the microscopic wavefunctions (i.e. the values of β). For the ^{28}Si(α, d)^{30}P reaction at E_α = 50 MeV the L-values ranged between 0 and 6 and the above requirements constrained the possible (r_0, a) values to area I in fig. 1. An additional restriction imposed by the well-matching prescription of Del Vecchio et al. [6] resulted

Table 1
Comparison between relative two-nucleon spectroscopic factors, G_L^2, calculated from the FPSDI interaction and the values A_L^2 obtained by fitting the calculated microscopic angular distributions with those calculated with a macroscopic form factor. The G^2 and A^2 are normalized to unity for the highest-J transfer.

E_x (MeV) a)	J^π a)	$(G_L^2)^{rel}$	$(G_{L+2}^2)^{rel}$	$(A_L^2)^{rel}_{macr}$	$(A_{L+2}^2)^{rel}_{macr}$
^{28}Si(α, d)^{30}P					
0	1^+	0.448	0.237	0.61 ± 0.04	0.29 ± 0.03
0.71	1^+	0.029	0.617	b)	0.82 ± 0.02
1.45	2^+	0.283	–	0.316 ± 0.005	–
1.97	3^+	0.041	6.1×10^{-4}	0.0370 ± 0.0004	<0.002
2.54	3^+	0.015	0.426	b)	0.66 ± 0.02
2.72	2^+	0.058	–	0.0583 ± 0.0007	–
2.84	3^+	0.007	0.334	b)	0.488 ± 0.015
3.02	1^+	9.7×10^{-4}	1.4×10^{-3}	$(9.2 \pm 0.8) \times 10^{-4}$	$(8.2 \pm 0.9) \times 10^{-4}$
7.20 c)	7^+ c)	1	–	1	–
^{32}S(d, α)^{30}P					
0	1^+	0.157	0.051	0.076 ± 0.005	0.048 ± 0.006
0.71	1^+	0.012	0.071	0.015 ± 0.004	0.057 ± 0.007
1.45	2^+	3.3×10^{-4}	–	$(2.10 \pm 0.14) \times 10^{-4}$	–
1.97	3^+	0.528	0.009	0.34 ± 0.04	0.10 ± 0.08
2.54	3^+	0.025	0.009	0.019 ± 0.002	0.014 ± 0.008
2.72	2^+	0.039	–	0.045 ± 0.005	–
2.84	3^+	0.045	0.024	0.039 ± 0.004	0.046 ± 0.015
3.02	1^+	0.028	0.012	0.0155 ± 0.0017	0.014 ± 0.002
4.34	5^+ c)	1	–	1	–
^{32}S(α, d)^{34}Cl					
0.15	3^+	0.005	0.485	b)	0.85 ± 0.02
0.46	1^+	0.035	0.207	0.062 ± 0.006	0.189 ± 0.010
0.67	1^+	0.044	0.048	0.046 ± 0.004	0.030 ± 0.005
1.23	2^+	0.028	–	0.0268 ± 0.0003	–
1.89	2^+	5.5×10^{-4}	–	$(3.78 \pm 0.07) \times 10^{-4}$	–
2.18	3^+	0.015	6.7×10^{-3}	$(9.0 \pm 0.2) \times 10^{-3}$	$(9.9 \pm 0.2) \times 10^{-3}$
2.38	4^+	3.1×10^{-4}	–	$(2.47 \pm 0.03) \times 10^{-4}$	–
2.58	1^+	5.5×10^{-6}	8.8×10^{-4}	b)	$(6.1 \pm 0.2) \times 10^{-4}$
2.61	3^+	1.9×10^{-3}	0.011	$(1.87 \pm 0.14) \times 10^{-3}$	0.0122 ± 0.0003
3.13	1^+	0.032	0.036	$(9.9 \pm 1.4) \times 10^{-3}$	0.037 ± 0.003
5.29 d)	7^+ d)	1	–	1	–

a) Taken from ref. [9] unless indicated otherwise.
b) The contribution of this L-value cannot be determined. The fitting procedure prefers a negative intensity. The A for the other L-value is determined by fitting with only one L-value.
c) Ref. [10]. d) Ref. [11].

in area II in fig. 1. From the overlap of the two areas we have fixed r_0 = 1.15 fm and a = 0.76 fm as effective values for all of the following macroscopic calculations. These values differ from the commonly used optical-model parameters [7] and indicate that one should be cautious by taking such parameters for the bound state. The corresponding areas for the ^{32}S(d, α)^{30}P and ^{32}S(α, d)^{34}Cl reactions include the above (r_0, a) combination.

Microscopic calculations, including the L–S potential and non-zero relative s-motions, in the angular range 0°–65° in steps of 1.67° have been taken as "data" to be compared with the macroscopic calculations. Both A_L^2 and A_{L+2}^2 have been determined in a χ^2-fitting procedure. Each "data" point was given an error of 1%.

Fig. 2 shows the fit of the macroscopic calculations to the microscopic "data". The shapes of the angular distributions are well reproduced, except at bakcward angles for the $J^\pi = 1^+$ states, for which L–S coherence effects probably play a role. The relative two-nucleon spectroscopic factors, G^2, calculated from the FPSDI shell-model wavefunctions [8] and those obtained in the fitting procedure, A^2, are presented in table 1. The uncertainties represent the maximum of the internal and external errors. The two sets of spectroscopic factors agree within about 25% and no systematic deviations occur. The differences may be due to non-zero relative s-motion and L–S coupling effects, which are included only in the microscopic calculations.

In conclusion, this procedure leads to macroscopic bound-state parameters which yield normalizations independent of L and almost independent of the microscopic wavefunctions. The deduced A^2 values may therefore be interpreted as relative two-nucleon spectroscopic factors. The macroscopic approach may thus be used to extract from data, analogously to SNT, spectroscopic information, which may be compared with the results from model calculations.

In a forthcoming paper [2] this method will be applied to extract spectroscopic factors from data on the ^{28}Si(α, d)^{30}P, ^{32}S(d, α)^{30}P and ^{32}S(α, d)^{34}Cl reactions and compared to the values for two sets of shell-model wavefunctions.

This work is part of the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM) with financial support of the Stichting voor Zuiver Wetenschappelijk Onderzoek (ZWO).

References

[1] J.C. Vermeulen et al., Nucl. Phys. A362 (1981) 189.
[2] R.J. de Meijer et al., submitted to Nucl. Phys.
[3] P.D. Kunz, University of Colorado, unpublished.
[4] B.F. Bayman and A. Kallio, Phys. Rev. 156 (1967) 1121.
[5] N.K. Glendenning, Phys. Rev. 137 (1965) 102.
[6] R.M. Del Vecchio and W.W. Daehnick, Phys. Rev. C6 (1972) 2095.
[7] C.M. Perey and F.G. Perey, At. Nucl. Data Tables 17 (1976) 1.
[8] B.H. Wildenthal, J.B. McGrory, E.C. Halbert and H.D. Graber, Phys. Rev. C4 (1971) 1708.
[9] P.M. Endt and C. van der Leun, Nucl. Phys. A310 (1978) 1.
[10] J.C. Vermeulen et al., Nucl. Phys. A329 (1979) 93.
[11] R.M. Del Vecchio, R.T. Kouzes and R. Sherr, Nucl. Phys. A265 (1976) 220.